

# Transitivity in Trust

A discussed property

Rino Falcone

Institute of Cognitive Sciences and Technologies  
National Research Council of Italy  
Rome, Italy  
rino.falcone@istc.cnr.it

Cristiano Castelfranchi

Institute of Cognitive Sciences and Technologies  
National Research Council of Italy  
Rome, Italy  
cristiano.castelfranchi@istc.cnr.it

**Abstract**—Transitivity in trust is very often considered as a quite simple property, trivially inferable from the classical transitivity defined in mathematics, logic, or grammar. In fact the complexity of the trust notion suggests evaluating the relationships with the transitivity in a more adequate way. In this paper, starting from a socio-cognitive model of trust, we analyze the different aspects and conceptual frameworks involved in this relation and show how different interpretations of these concepts produce different solutions and definitions of trust transitivity.

**Keywords-component; Trust, Transitivity, Degree of Trust, Task definition**

## I. INTRODUCTION

Trust is becoming a research topic of major interest not only in Artificial Intelligence (AI) and in Multi-Agent Systems (MAS), but also more in general in Information and Communication Technologies (ICT). The main reason of this fact is that the more recent developments of the “interaction” paradigm of computation, are driving more and more towards the development of computational entities with a strong and well defined autonomy. These entities have to cooperate/conflict among them in conditions of open world for achieving their own goals.

In perspective, we are going towards an interaction scenario in which artificial entities and humans are indistinguishable from each other. In this view, the probability that we have to interact or cooperate with entities we do not have any personal experience with, will be growing, and the need of attributing *trustworthiness* to the potential partners becomes a fundamental prerequisite. And to model trust in the way in which humans have always done it, being them in the interaction loop, is particularly relevant.

Many different approaches and models of trust were developed in the last 15 years [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]: they contributed to clarify many aspects and problems about trust and trustworthiness, although many issues remain to be addressed and some elementary but not so trivial trust properties are left in a contradictory form.

One of them is the problem of trust transitivity. If  $X$  trusts  $Y$ , and  $Y$  trusts  $Z$ : What about the trust relationship between  $X$  and  $Z$ ? Different and sometimes diverging answers were given to this problem. The question is not only theoretically

relevant; it is very relevant from the practical point of view, for the reason we have just mentioned: acting in an open world, interacting with new people/agents.

In this paper we will present an analysis of the trust transitivity in the case in which a socio-cognitive model of trust is taken in consideration. Through this kind of model we are able to evaluate and partially cope with the complexity that the concept of transitivity introduces when applied to the trust relationship.

## II. A SOCIO-COGNITIVE MODEL OF TRUST

In our socio-cognitive model of trust [11, 12, 13, 14] we consider trust as a layered notion, where the various more or less complex meanings are embedded one into the other. We analyzed the relevant relationships among these different layers and studied the possible transitions among them.

We developed the analysis of the *mental attitude* and *disposition* towards the trustee (considering beliefs like *evaluations* and *expectations*); the *intention* and *decision* based on the previous dispositions; the *act* of relying upon the trustee’s expected behaviour; finally the *social interaction* and *relation* between trustor and trustee.

In particular we consider trust as a relational construct between the trustor ( $X$ ), the trustee ( $Y$ ), about a defined (more or less specialized) task ( $\tau$ ):

$$\text{Trust}(X Y C \tau g_X)$$

where are also explicitly present both the  $X$ ’s goal ( $g_X$ , respect to which trust is activated) and the role of the context ( $C$ ) in which the relationship is going to happens. In fact, the successful performance of the task  $\tau$  will satisfy the goal  $g_X$ .

So the  $X$ ’s mental ingredients of trust are: the goal  $g_X$ , and a set of *main* beliefs:

$$\text{Bel}(X \text{ Can}_Y(\tau))$$

$$\text{Bel}(X \text{ Will}_Y(\tau))$$

$$\text{Bel}(X \text{ ExtFact}_Y(\tau))$$

where:

$\text{Can}_Y(\tau)$  means that  $Y$  is potentially able to do  $\tau$  (in the sense that, under the given conditions, is competent, has the internal powers, skills, know-how, etc) (and this is believed by  $X$ );

### III. TRANSITIVITY IN TRUST

*Will<sub>Y</sub>(τ)* means that, under the given conditions, *Y* potentially has the attributions for being willing, persistent, available, etc., on the task *τ* (and this is believed by *X*);

*ExtFact<sub>Y</sub>(τ)* means that potentially there are a set of external conditions either favoring or hindering *Y* realizing the task *τ* (and this is believed by *X*).

In our model we also consider that trust can be *graded*: *X* can have a *strong trust* that *Y* will realize the task (maybe 0.95 in the range (0,1)); or even just a *sufficient trust* that *Y* will achieve it (maybe 0.6 with a threshold of 0.55; and so on with other possible values). For this we have introduced a quantification of the degree of trust (*DoT<sub>XYτ</sub>*) and, in general, a threshold (*σ*) to be overcome from this *DoT<sub>XYτ</sub>*.

Given the previous analysis of the main components of the trust attitude (*g<sub>X</sub>*, *Bel(X Can<sub>Y</sub>(τ))*, *Bel(X Will<sub>Y</sub>(τ))*, *Bel(X ExtFact<sub>Y</sub>(τ))*), we can say that this degree is, on its turn, resultant from the several *quantifications* of these components:

$DoT_{XY\tau} = f(DoC_X(Opp_Y(\tau)), DoC_X(Abil_Y(\tau)), DoC_X(Will_Y(\tau)))$   
where: *f* is in general a function that preserves monotonicity;

*DoC<sub>X</sub>(Opp<sub>Y</sub>(τ))* is the *X*'s degree of credibility about the external opportunities (positively or negatively) interfering with *Y*'s action in realizing the task *τ*;

*DoC<sub>X</sub>(Abil<sub>Y</sub>(τ))* is the *X*'s degree of credibility about the *Y*'s ability/competence to perform *τ*;

*DoC<sub>X</sub>(Will<sub>Y</sub>(τ))* is the *X*'s degree of credibility about the *Y*'s willingness to perform *τ*.

We are ignoring the subjective certainty of the pertinent beliefs (how much sure is *X* of its evaluative beliefs about that specific *Y*'s quality, that is a meta-belief; in fact we can say that this factor is integrated with the other). At the same time we are ignoring for now the value of the goal (*g<sub>X</sub>*).

So trivially *X* will trust *Y* about the task *τ* if

$$DoT_{XY\tau} > \sigma$$

that means that a set of analogous conditions must be realized about the other quantitative elements (*DoC<sub>X</sub>(Opp<sub>Y</sub>(τ))*, *DoC<sub>X</sub>(Abil<sub>Y</sub>(τ))*, *DoC<sub>X</sub>(Will<sub>Y</sub>(τ))*). We do not consider in this paper the detailed analysis of how the degree of trust is resulting by the more elementary components and we also omit of considering the potential positive and negative interferences among the components themselves.

Introducing also the concept of *Y*'s *trustworthiness degree* (*Trustworthiness<sub>YX</sub>(τ)*) with respect to *X* about the task *τ*, we can say that from:

$$DoT_{XY\tau} > \sigma$$

it derives that

$$Bel(X Trustworthiness_{YX}(\tau) > \Sigma)$$

In general  $\sigma = \Sigma$ .

Many authors have questioned whether the transitive property can be applied to trust. In more specific words many of them have presented this problem in this way:

If *X* trusts *Y*, and *Y* trusts *Z*: What about the trust relationship between *X* and *Z*?

Their answers are different and very briefly we will analyze some of them in §IV.

We are now interested to translate this problem in our terms of trust.

First of all, we do not consider the unspecified case “*X* trusts *Y*” because in our model an agent has to trust another agent with respect a task (either very well defined or less defined and abstract); this task directly derives from the goal the trustor has to reach with the trust attribution. So we have to transform “*X* trusts *Y*” in “*X* trusts *Y* about *τ*”. And given the graded qualification of trust we have that:

$$DoT_{XY\tau} > \sigma$$

this means in particular that *X* believes that *Y* is potentially able and willing to do *τ* and that the external conditions in which *Y* will perform its task are at least not so opposite to the task realization (may be also they are neutral or favorable).

So this *Y*'s trustworthiness with respect to *X* (perceived/believed by *X*) is based on these specific beliefs.

At the same way “*Y* trusts *Z*” becomes “*Y* trusts *Z* about *τ<sub>1</sub>*” (about the difference between *τ* and *τ<sub>1</sub>*, see later) with the same particular *Y*'s beliefs about *Z* and the external conditions.

Also in this case we can say that there is a threshold to be overcome and the condition:

$$DoT_{YZ\tau_1} > \sigma_1$$

successfully satisfied in case of trust attribution.

If we have to consider the trust relationship between “*X* and *Z*” as a consequence of the previous trust relationships between “*X* and *Y*” and between “*Y* and *Z*” we have to define the task on which this relationship should be based (question of *assimilation* between *τ* and *τ<sub>1</sub>*) and the degree of trust that must be overcome even from *X*:

$$DoT_{XZ\tau} > \sigma_2$$

with the consideration of the threshold *σ<sub>2</sub>*.

The role of the trust threshold is quite complex and can have an overlapping with the ingredients of trust. We strongly simplify in this case considering *σ* as dependent just from the specific intrinsic characteristics of the trustor (those that define an agent intrinsically: prudent, reckless, and so on) independently from the external circumstances (on the contrary, these factors affect the degree of trust, by affecting the more elementary beliefs above showed).

So, we can say that in this approximation (for the same agent the trust threshold is always the same):

$$\sigma = \sigma_2$$

In the case in which all the agents are defined as having the same intrinsic characteristics (this fact is possible in the case of artificial entities), we can also say that:

$$\sigma = \sigma_1 = \sigma_2$$

Moreover, as we just saw, not less important in our approach is that trust is an expectation and a bet *grounded on and justified by* certain beliefs about  $Y$ .  $X$  trusts  $Y$  on the basis of the evaluation of  $Y$ 's "virtues/qualities", not just on the basis of a statistical sampling, some probability.

The *evaluations* about the needed "qualities" of  $Y$  for  $\tau$  are the *mediator* for the decision to trust  $Y$ . This mediation role is fundamental also in trust transitivity.

Let us now consider the case of the differences between the tasks in the different relationships.

For the trust transitivity the two tasks should be the same ( $\tau = \tau_1$ ). Is this equality enough?

Suppose for example that there are 3 agents: John, Mary and Peter; and suppose that John trusts Mary about "organizing scientific meetings" (task  $\tau$ ), at the same time Mary trusts Peter about "organizing scientific meetings" (again task  $\tau$ ). Can we deduce that, given the transitivity of trust: John trusts Peter about "organizing scientific meetings"? Is in fact transferable that task evaluation? Given the trust model defined in §2 the situation is more complex and there are possible pitfalls lurking: Mary is the central node for that trust transfer and she plays different roles (and functions) in the first case (when her trustworthiness is about *to realize* the task  $\tau$ , and in the second case (when her trustworthiness is about *evaluating* the Peter's trustworthiness on the task  $\tau$ ).

The situation is even clearer if we split in the example the two kinds of competences:  $X$  trusts very much  $Y$  as medical doctor;  $X$  knows that  $Y$  trusts  $Z$  as mechanic; will  $X$  trust  $Z$  as mechanic? Not necessarily at all: if  $X$  believes that  $Y$  is a good evaluator of mechanics he will trust  $Z$ ; but, if  $X$  believes that  $Y$  is a very naive in this domain, and is frequently swindled by people, he will not trust  $Z$ . In the previous example the transition looks more plausible, natural, just because the task  $\tau$  is the same, and it is reasonable (but not necessary) that if  $Y$  is very skilled and competent in task  $\tau$ , she will also be a good evaluator of other people on the same task.

So for considering transitivity of trust as a valid property (in the classical way in which it is defined) in these types of situations, we have to *assimilate* the task with the evaluation of that task itself:

$$Bel(X \text{ Trustworthiness}_{YX}(\tau) > \sigma)$$

**implies**  $Bel(X \text{ Trustworthiness}_{YX}(eval(\tau)) > \sigma)$

In words,  $X$  has to believe that if  $Y$  is trustworthy on the task  $\tau$ , it is also trustworthy on the meta-task of evaluating  $\tau$  (in both the situations the  $X$ 's mental ingredients defining the trust in  $Y$  allow to overcome the threshold).

We do not consider in this paper the truthfulness of this hypothesis (and the consequent properties both in the more elementary mental ingredients of the interacting actors, and in the tasks' features): in fact, our trust model is apt to analyze in detail this problem. However, we want to underline the difference of the involved tasks in the relationships and the necessity of taking into consideration these differences before generally speaking of trust transitivity.

So resumming we have the *more basic case* of the relationship between *trust* and *transitivity* so defined (case A):

**if**  
 iA)  $DoT_{XY\tau} > \sigma_X$  ( $X$  trusts  $Y$  about  $\tau$ ) and  
 iiA)  $DoT_{YZ\tau} > \sigma_Y$  ( $Y$  trusts  $Z$  about  $\tau$ ) and  
 iiiA)  $Bel(X \text{ DoT}_{YZ\tau} > \sigma_Y)$  ( $X$  believes that  $Y$  trusts  $Z$  about  $\tau$ )  
 and  
 ivA)  $Bel(X \text{ Trustworthiness}_{YX}(\tau) > \sigma_X)$  **implies**  
 $Bel(X \text{ Trustworthiness}_{YX}(eval(\tau)) > \sigma_X)$   
 ( $Y$  is equally trustworthy in the realization of the task and in evaluating others performing that task; this implication also takes into account the fact that  $X$  has a good consideration about the adopted threshold by  $Y$ ,  $\sigma_Y$ )  
**then**  
 vA)  $DoT_{XZ\tau} > \sigma_X$  ( $X$  trusts  $Z$  about  $\tau$ )

In the case (B) in which we assume  $\sigma = \sigma_X = \sigma_Y$  (the trust threshold is the same for each agent and for each relation), we have:

**if**  
 iB)  $DoT_{XY\tau} > \sigma$  ( $X$  trusts  $Y$  about  $\tau$ ) and  
 iiB)  $DoT_{YZ\tau} > \sigma$  ( $Y$  trusts  $Z$  about  $\tau$ ) and  
 iiiB)  $Bel(X \text{ DoT}_{YZ\tau} > \sigma)$  ( $X$  believes that  $Y$  trusts  $Z$  about  $\tau$ )  
 and  
 ivB)  $Bel(X \text{ Trustworthiness}_{YX}(\tau) > \sigma)$  **implies**  
 $Bel(X \text{ Trustworthiness}_{YX}(eval(\tau)) > \sigma)$   
 ( $Y$  is equally trustworthy in the realization of the task and in evaluating others performing that task)  
**then**  
 vB)  $DoT_{XZ\tau} > \sigma$  ( $X$  trusts  $Z$  about  $\tau$ )

The fact that are true:  $DoT_{XY\tau} > \sigma$ ,  $DoT_{YZ\tau} > \sigma$ , and  $DoT_{XZ\tau} > \sigma$

does not mean that necessarily  $DoT_{XY\tau} = DoT_{YZ\tau} = DoT_{XZ\tau}$ . As we have seen in the §2 these degrees are dependent from the internal beliefs on the different components, and they are resulting from different sources not necessarily all common to the involved agents.

We have also to underline that (iiA) or (iiB) should not necessarily be true, the import thing is that are respectively true (iiiA) or (iiiB).

In the case (C) in which the *tasks are different* ( $\tau \neq \tau'$ ), we have (we are also assuming that  $\sigma = \sigma_X = \sigma_Y$ ):

**if**  
iC)  $DoT_{XY\tau} > \sigma$  ( $X$  trusts  $Y$  about  $\tau$ ) and  
iiC)  $DoT_{YZ\tau} > \sigma$  ( $Y$  trusts  $Z$  about  $\tau$ ) and  
iiiC)  $Bel(X DoT_{YZ\tau} > \sigma)$  ( $X$  believes that  $Y$  trusts  $Z$  about  $\tau$ )  
and  
ivC)  $Bel(X Trustworthiness_{YX}(\tau) > \sigma)$  **implies**  
 $Bel(X Trustworthiness_{YX}(eval(\tau')) > \sigma)$   
( $Y$  is equally trustworthy in the realization of the task and in evaluating others performing that specific different task)  
**then**  
vC)  $DoT_{XZ\tau'} > \sigma$  ( $X$  trusts  $Z$  about  $\tau'$ )

In this last case C, the transitivity essentially depends from the implication reported in the formula (iiiC); are there elements in the reasons (believed by  $X$ ) for trusting  $Y$  on the task  $\tau$  that (in  $X$ 's view) are sufficient also for trusting  $Y$  on the different task  $\tau'$ ?

#### A. Trust and Transitivity in the delegated subtasks

Another interesting case of the relationship between trust and transitivity is when we have the following situation:

“ $X$  trusts  $Y$  about  $\tau$ ” and, in realizing the task ( $\tau$ ),  $Y$  delegates parts of the task itself to other agents  $Z$ ,  $W$  (for example the delegated subtasks are respectively  $\tau_1$  and  $\tau_2$ ).

Then if we suppose that  $X$  is aware of this delegation; what we can say (on the basis of the trust relationship between  $X$  and  $Y$ ) about the trust between  $X$  and  $Z$  with respect to  $\tau_1$ ? And between  $X$  and  $W$  with respect to  $\tau_2$ ?

Suppose, for example, that John trusts Mary about “organizing a scientific meeting” and that Mary delegates Peter to “organize the registration process”, and delegates Alice to “organize the sponsoring of the event”. What about the John’s direct trust on Peter (as organizing the registration process) and the John’s direct trust on Alice (as organizing the sponsoring of the event)? Are these trust relationships the same of the Mary’s ones? How are they *mediated* by the John’s trust on Mary?

We have that:

$$DoT_{John\ Mary\ \tau} > \sigma \text{ and}$$

$$DoT_{Mary\ Peter\ \tau_1} > \sigma \text{ and}$$

$$DoT_{Mary\ Alice\ \tau_2} > \sigma$$

But from these assumptions does not necessarily follow that:

$$DoT_{John\ Peter\ \tau_1} > \sigma \text{ and/or } DoT_{John\ Alice\ \tau_2} > \sigma$$

In fact, John should know how exactly the delegation of the subtasks is realized and on what basis is founded. In a trust relationship, as we have seen in §2, are involved not only qualities about abilities and skills but also qualities about willingness, availability, and so on. So these others qualities could be elicited by the specific relation with the trustor (delegating agent) and in some cases are strictly related with the interaction history among the agents (see §3.2 for a more detailed analysis of this). In these cases is more difficult for

John to evaluate which could be the Peter’s and Alice’s performances (and then their trustworthiness).

In addition may be there is a particular interaction among the subtasks and the main task in which Mary plays a specific role of integration and substitution (in presence of shortcomings of the other agents (Peter and Alice)) that is essential for the success of the complete task. Also in this case John trusting Peter and Alice, at the same way of Mary, should be aware to be able of playing (in case of necessity) the same role of integration and substitution.

So we can say that for applying the trust transitivity to the cases of subtasks delegation, we have to analyze in deep detail: on the one hand the set relationships among task and its subtasks (and how they are distributed among the agents in play), and, on the other hand, how the executing agents are *motivated* and *activated* in the task realization by the relationship with the trustor.

Resuming in the case (D) of *delegated subtasks* we can say that:

**if**  
iD)  $DoT_{XY\tau} > \sigma$  and  
iiD)  $DoT_{YZ\tau_1} > \sigma$  where  $\tau_1$  is a subtask of  $\tau$  (realizing  $\tau_1$  are achieved parts of the result of  $\tau$ ) and  
iiiD)  $Bel(X DoT_{YZ\tau_1} > \sigma)$  and  
ivD)  $DoT_{YW\tau_2} > \sigma$  where  $\tau_2$  is a subtask of  $\tau$  (realizing  $\tau_2$  are achieved parts of the result of  $\tau$ ) and  
vD)  $Bel(X DoT_{YW\tau_2} > \sigma)$  and  
viD)  $Bel(X Trustworthiness_{ZX}(\tau_1) \geq Trustworthiness_{ZY}(\tau_1))$  and  
viiD)  $Bel(X Trustworthiness_{WX}(\tau_2) \geq Trustworthiness_{WY}(\tau_2))$   
**then**  
viiiD)  $DoT_{XZ\tau_1} > \sigma$  and  $DoT_{XW\tau_2} > \sigma$ .

#### B. Competence and Willingness in Transitivity

The need for a careful qualitative consideration of the nature of the link between the trustor and the trustee, is even more serious.

Not only it is fundamental (as we have argued) to make explicit and do not forget the specific “task” (activity, and thus “qualities”)  $X$  is trusting  $Y$  or  $Z$  about, but it is even necessary to consider the different dimensions/components of the trust disposition (evaluation), decision, and relation. In our model (a part from the basic thought and feeling that I have not to worry about  $Y$ , that there is no danger from  $Y$ 's side, that I do not need diffidence and a defensive attitude), trust has two basic nucleuses:

- (i)  $Y$ 's *competence*, ability, for correctly performing the delegated task;
- (ii)  $Y$ 's *willingness* to do it, to act as expected.

The two dimensions (and ‘virtues’ of  $Y$ ) are quite independent on each other:  $Y$  might be very well disposed and willing to do, but not very competent or unable;  $Y$  might be very expert and skilled, but not very reliable: unstable, unpredictable, not well disposed, insincere, dishonest, etc.

Now, this (at least) double dimensions affect transitivity. In fact, even assuming that the *competence* is rather stable (see below) (and that  $Y$  is a good evaluator of  $Z$ 's *competence*) not necessarily  $Z$ 's *willingness* is equally stable and transferable from  $Y$  to  $X$ . This is a more relation-based dimension.  $Y$  was evaluating  $Z$ 's *willingness* to do as expected on the basis of their specific *relation*. Is  $Z$  a friend of  $Y$ ? Is there a specific benevolence, or values sharing, or gratitude and reciprocity, or obligation and hierarchical relation, etc.? Not necessarily the reasons that  $Z$  would have for satisfying  $Y$ 's expectation and delegation would be present (or equally important) towards  $X$ .  $X$ 's relation with  $Z$  might be very different. Are the reasons/motives motivating  $Z$  towards  $Y$ , and making him reliable, transferable or equally present towards  $X$ ? Only in this case it would be reasonable for  $X$  to adopt  $Y$ 's trustful attitude and decision towards  $Z$ . Only certain kinds of relations will be generalized from  $Y$  to  $X$ ; for example, if  $Y$  trusts  $Z$  only because it is an economic exchange, only for  $Z$ 's interest in money, reasonably  $X$  can become a new client of  $Z$ ; or if  $Y$  relies on  $Z$  because  $Z$  is a charitable person, generously helping (without any prejudice and discrimination) poor suffering people, and  $X$  is in the same condition of  $Y$ , than also  $X$  can trust in  $Z$ . In sum,

- $Y$ 's expectation about  $Z$ 's reliable behavior, in particular about  $Z$ 's “adoption” of  $Y$ 's goal (help, etc.) *depends on the relationship between  $Y$  and  $Z$* , and in particular on  $Z$ 's *attitude towards  $Y$*  (and reasons for goal-adoption); if the *relation between  $X$  and  $Z$* , and in particular  $Z$ 's *attitude towards  $X$*  (and reasons for goal-adoption), would be analogous, then the trust “transfer” would be reasonable.

In the previous analysis and examples we have partially solved this problem considering the concept of  $Y$ 's trustworthiness towards  $X$  about the task  $\tau$ , in which the trustworthiness of the trustee ( $Y$ ) also depends from the trustor ( $X$ ) and the task ( $\tau$ ).

### C. Trust Dynamics affects Transitivity

Moreover, we have shown ([13], [14]) that  $Z$ 's willingness, and even ability, can be affected, increased, by  $Y$ 's trust and reliance (this can affect  $Z$ 's commitment, pride, effort,... attention, study,...).  $Z$ 's *trustworthiness* is improved by  $Y$ 's trust and delegation. And  $Y$  might predict and calculate this in her decision to rely on  $Z$ .

However, not necessarily the effect of  $Y$  on  $Z$ 's trustworthiness will be produced also by another trustor. Thus, also this will affect “transitivity”: suppose that  $Y$ 's trust and delegation to  $Z$  makes him more trustworthy, improves  $Z$ 's willingness or ability (and  $Y$  trusts and relies on  $Z$  on the basis of such expectation); not necessarily  $X$ 's reliance on  $Z$  would

have the same effect. Thus even if  $X$  knows that  $Y$  reasonably trusts  $Z$  (for something) and that he is a good evaluator and decision-maker, not necessarily  $X$  can have the same trust in  $Z$ , since perhaps  $Z$ 's trustworthiness would not be equally improved by  $X$ 's reliance.

Trust 'dynamics' between  $Y$  and  $Z$ , is not automatically identical to trust “dynamics” between  $X$  and  $Z$ .

## IV. TRANSITIVITY AND TRUST: RELATED WORK

The necessity of modeling trust in the social networks is becoming more and more important, and a set of new definitions are emerging in different domains of computing: *computational trust*, *trust propagation*, *trust metrics*, *trust in web-based social networks*, and so on. Most of these concepts are strictly linked with the goal of inferring trust relationships between individuals that are not directly connected in the networks. For this reason the concept of trust transitivity is very often considered and used in these approaches.

A relevant example is given from the Josang's approach; he introduces the subjective logics (an attempt of overcoming the limits of the classical logics) for taking in consideration the uncertainty, the ignorance and the subjective characteristics of the beliefs [15]. Using this approach Josang addressed the problem of trust transitivity in different works [16], till the last developments, see [17], where it is recognized the intrinsic cognitive nature of this phenomenon. However, the main limits of this approach are that trust is in fact the *trust in the information sources*; and the transitivity regards two different tasks (referred to our formalism:  $\tau \neq \tau_1$ :  $X$  has to trust the evaluation of  $Y$  (task  $\tau$ ) with respect  $Z$  as realizing another task (task  $\tau_1$ , for example as mechanic). As we showed before, this difference is really relevant for the transitivity phenomenon. In addition, also the first task ( $Y$  as evaluator) is just analyzed with respect to the *property of sincerity* (and this is a confirmation of the constrained view of trust phenomenon; they write: “ $A$ 's disbelief in the recommending agent  $B$  means that  $A$  thinks that  $B$  consistently *recommends the opposite* of his real opinion about the truth value of  $x$ ”; where  $A$ ,  $B$ , and  $x$  are, in our terms, respectively  $X$ ,  $Y$  and  $\tau_1$ ). But in trusting someone as evaluator of another agent (with respect to a specific task), I have also to consider his *competence* as evaluator, not just his *sincerity*. Trust is based on ascribed qualities.  $Y$  could be completely sincere but at the same time completely inappropriate to evaluate that task. So the limits of this approach of an adequate treatment of the trust transitivity are quite clear.

Many other authors [18, 19], developed algorithms for inferring trust among agents not directly connected. These algorithms differ from each other in the way they compute trust values and propagate those values in the networks. In any case when trust transitivity is introduced, this phenomenon is not analyzed with respect the complexity it contains and that we tried to explain in the previous paragraphs.

## CONCLUSIONS

The relationships between Transitivity and Trust (like transitivity in partial order and equivalence relations) represent an important element to well-understand the intimate nature of the trust concept. Since trust is considered the glue of social interactions, its complex nature has to be deeply investigated if we want realize a careful and precise model to be transferred in the artificial societies. The analysis of the trust properties is, in this sense, a very useful tool. And transitivity (among the other properties) is one of the more interesting one. In this paper we analyzed the relationships between trust and transitivity, showing in particular, how this analysis implicates the evaluation of the tasks involved in the different relationships, of the qualities of the agents in play in those specific tasks and of the particular relationships among the agents (and of their interaction histories and contexts).

We tried to show how a too trivial simplification of the transitive property when applied to the trust concept leads to wrong conclusions about the model of the phenomenon we are studying.

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